

UNIT-I: MATRICES

1. Algebra of Matrices

Matrix; Types of Matrices; Equal Matrices; Addition of Matrices; Properties of Matrix Addition; Scalar Multiplication of a Matrix; Multiplication of Matrices; Properties of Matrix Multiplication; Positive integral power of a square matrix; Zero Divisors; Illustrative Examples; Exercise (A); Transpose of a Matrix; Some Theorems on Transposed Matrix; Skew-symmetric matrix; Conjugate of a Matrix; Hermitian Matrix; Skew hermitian Matrix; Some Theorems; Illustrations Examples; Exercise (B); Idempotent Matrix; Nilpotent Matrix; Involutory Matrix; Orthogonal Matrix; Unitary matrix; Singular matrix; Some Theorems; Illustrative Examples; Exercise (C); Some Theorems; Illustrative Examples; Exercise (D); Inverse of a Matrix; Some Theorems on Inverse of a Matrix; Illustrative Examples; Exercise 1 (E).

2. Elementary Operations

Elementary Operations; Elementary Matrices; Some Theorems on Elementary Operations; Equivalent Matrices; Computation of the inverse of a matrix by elementary operations; Illustrative Examples; Exercise 2 (A); Triangular Matrix; Reduction of Matrices to triangular form; Partitioning of Matrices; Identically Partitioned Matrices; Matrices Partitioned Conformably for Multiplication; Illustrative Examples; Exercise 2 (B).

3. Rank of a Matrix

Minor of a Matrix; Rank of a Matrix; Nullity of a Matrix; Illustrative Examples Exercise 3 (A); Normal Form; Some Theorems on Rank; Illustrative Examples Exercise 3 (B); Zero row and non-zero row; Echelon form of a Matrix; Rank of a matrix in Echelon form; Illustrative Examples; Exercise 3 (C).

4. Linear Equations

Linear Equations; Solution of a system of a non-homogeneous linear equations in n unknowns; Solution of m non-homogeneous linear equations in n variables; Illustrative Examples; Exercise 4(A).

5. Linear Dependence of Vectors

Ordered set of numbers; Vectors; Algebra of Vectors; Linear dependence and linear independence of Vectors; Linear combination of Vector; Illustrative Examples; Some Basic Theorems; Illustrative Examples; Exercise 5.

6. Characteristic Roots and Vectors

Characteristic Value Problem; Some Fundamental Theorems; Illustrative Examples; Exercise 6 (A); Cayley–Hamilton Theorem; Computation of the inverse by Cayley–Hamilton Theorem; Exercise 6 (B); Similarity of Matrices; Diagonalizable matrix; Illustrative Examples; Illustrative Examples.

UNIT-II : DIFFERENTIAL CALCULUS—I**7. Successive Differentiation**

Successive Differential Coefficients; Exercise 7 (A); Calculations of n th derivatives; Illustrative Examples; Exercise 7 (B); Leibnitz's Theorem; Illustrative Examples; Exercise 7 (C); Use of Leibnitz's Theorem; Illustrative Examples; Exercise 7 (D); Illustrative Examples; Exercise 7 (E).

8. Partial Differentiation

Introduction; Partial derivatives of $f(x, y)$; Partial derivatives and continuity of $f(x, y)$; Illustrative Examples; Differentiability of the function $f(x, y)$; Mean-Value Theorem for a function $f(x, y)$; Necessary conditions for differentiability; Sufficient Condition for Differentiability; Higher Partial derivatives; Equality of Mixed partial derivatives; Schwarz's Theorem; Young's Theorem; Illustrative Examples; Exercise 8 (A); Homogeneous Functions; Euler's Theorem; Illustrative Examples;

Exercise 8 (B); Total Differential coefficients or Total Derivatives; To find $\frac{d^2y}{dx^2}$ from $f(x, y) = c$; Illustrative Examples; Exercise 8 (C).

9. Curve Tracing

Introduction; Tracing of cartesian curves; Illustrative Examples; Exercise 9 (A); Tracing of polar Curves; Illustrative Examples; Exercise 9 (B); Tracing of Parametric Equations; Illustrative Examples; Some Important Curves.

10. Transformation of Coordinates

Transformation from Cartesian to Polars coordintes and vice-versa; x and y to be expressed in terms of some third variable; Illustrative Examples; Exercise 10(A) Two Independent Variables; Transformation from cartesian to the polar

Transformation of $\frac{\partial^2 V}{\partial x^2}$ and $\frac{\partial^2 V}{\partial y^2}$ to polars ; Transformation of $\nabla^2 V$

Illustrative Examples; Exercise 10 (B).

11. Expansion of Functions and Several Variables

Taylor's Theorem (for two variables); Other Form; Taylor's Theorem for three variables; Maclaurin's theorem for function of two variables; Maclaurin's theorem for function of three variables; Illustrative Examples; Exercise 11.

UNIT-III : DIFFERENTIAL CALCULUS—II

12. Jacobians

Definition; Particular case; Illustrative Examples; Exercise 12 (A); Jacobian of Function of Function; Jacobian of Implicit Functions; Particular case; Illustrative Examples; Exercise 12 (B); Theorem; Illustrative Examples; Exercise 12 (C).

13. Approximation and Errors

Approximations and Errors; Approximate Calculations in two or more Variables; Illustrative Examples; Exercise 13.

14. Maxima and Minima

Definition of a Maximum or Minimum; Graphical Concept; A necessary condition for maxima or minima; Maxima and minima of functions of two variables; Condition for the existence of Maxima or Minima; Stationary and extreme Points; Sign of quadratic expressions : Algebraic Lemma; Lagrange's condition for maximum and minimum values of a function of two variables; Lagrange's condition for Three Independent Variables; Illustrative Examples; Exercise 14 (A); Lagrange's Method of Undetermined Multipliers (for several independent variables); Illustrative Examples; Exercise 14 (B).

UNIT-IV : MULTIPLE INTEGRALS

15. Multiple Integrals

Multiple Integrals; Double Integral; Working Method; Double Integration for polar curves; Illustrative Examples; Exercise 15 (A); Change of order of Integration; Illustrative Examples; Exercise 15 (B); Change of the variable in a multiple integral; Illustrative Examples ; Exercise 15 (C); Illustrative Examples ; Exercise 15 (D); Illustrative Examples; Application of Double Integration; Illustrative Examples; Exercise 15 (E); Application of Multiple Integrals to find C.G. when density varies; Illustrative Examples; Exercise 15 (F); General formulae for the centre of gravity of any volume; Applications of Multiple integrals to find Moment and Products of Inertia; Illustrative Examples; Exercise 15 (G).

16. Beta and Gamma Functions

5

Definitions; Properties of Beta and Gamma Functions; Transformations of Beta Functions; Relation between Beta and Gamma Functions; To prove that

$$\int_0^{\pi/2} \sin^m \theta \cos^n \theta d\theta = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{m+n+2}{2}\right)}; \text{ Duplication Formula; Illus-}$$

trative Examples; Exercise 16 (A); Evaluate the Integrals; Illustrative Examples; Exercise 16 (B).

17. Dirichlet's Theorem

5

Dritchlet's Theorem; Liouville's extension of Dritchlet's Theorem; Illustrative Examples; Exercise 17 (A).

UNIT-V : VECTOR CALCULUS

18. Differential Operators

5

Partial Derivatives; Illustrative Examples; Exercise 18 (A); Scalar Point Function; Vector point Function; Gradient or Slope of a Scalar Point-function.; Total differential df , where f is a scalar point function.; Theorem; Gradient in Polar Coordinates; Illustrative Examples; Exercise 18 (B); Scalar and Vectors Fields; Equipotential or Level Surfaces; Directional Derivative of a Function; Some Theorems; Tangent Plane and Normal Line; Tangent Line and Normal Plane; Illustrative Examples; Exercise 18 (C); Divergence of a Vector; Curl of a Vector; The Laplacian operator ∇^2 ; Illustrative Examples; Exercise 18 (D); Illustrative Examples; Exercise 18 (E); Properties of second order differential operators; Illustrative Examples; Exercise 18 (F); Vector Identities; Illustrative Examples; Exercise 18 (G); Physical Significance of $\nabla\phi$.

19. Line and Surface and Volume Integrals

6

Line Integrals; Illustrative Examples; Exercise 19 (A); Normal Surface Integral; Volume Integral; Illustrative Examples; Exercise 19 (B); Circulation; Line Integral of $\text{grad } \phi$; Irrotational Vector Field; Condition for Irrotational Vector Field; Work done by a force field; Conservative Fields; Some Theorems on conservative fields; Illustrative Examples; Exercise 19 (C).

20. Gauss's Green's and Stoke's Theorem

6

Gauss's Divergence Theorem; Illustrative Examples; Exercise 20 (A); Green's Theorem in the Plane; Illustrative Examples; Exercise 20 (B); Stokes Theorem; Stokes Theorem in Cartesian Form; Illustrative Examples; Exercise 20 (C).